

A Path to the *Tractatus*: From Facts and Forms through Picturing to Modality

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This paper traces Wittgenstein's philosophical development culminating in the conception, central to *Tractatus Logico-Philosophicus* (1922: cited by remark number), of propositions as involving essentially a primitive notion of possibility. In §§2-7, I spell out three stages of Wittgenstein's philosophical development, starting from criticism of Russell's multiple-relation theory of judgment, going through the conception of propositions as signifying facts in "Notes on Logic," to struggles with what Wittgenstein calls the "Wahrheitsprobleme" in his wartime notebooks, in which he first considers appealing to the idea of picturing to understand propositions, before finally arriving at the modal view of proposition in the *Tractatus*. §1 outlines the philosophical context that illuminates the significance of the Tractarian conception of propositions: Frege and Russell's rejection of modality, and so of the ancient tradition of characterizing logic in modal terms. Against this background, Wittgenstein's view of propositions in the *Tractatus* appears as part of a rejoinder to Frege and Russell, bringing modality back into logic.

1. *Modality Lost*

A long tradition, starting with Aristotle, characterizes logic in term of modality: an argument is deductively valid if, as Aristotle put it, its conclusion follows "out of necessity" Aristotle (1938: 24b18-20) from its premises, in other words, if it's impossible for the conclusion to be false and the premises true. Frege and Russell reject this tradition, at least if it's understood as taking deductive validity to be explained in terms of the more basic notions of necessity and possibility. According to Frege, "calling a statement necessary *has no meaning for us*" (1879: §4, 5; emphases in original). Russell is less diplomatic: "Modality ought to be banished from logic" (1905: 520).

Why did Frege and Russell hold these views about modality? I suspect that we are strongly tempted to think our way to an answer as follows. We take Frege and Russell very seriously. We take modality very seriously. So, Frege and Russell's rejection of modality must not be an important part of their philosophies. It's probably just "something in the air," in the *Zeitgeist*: after all, didn't Lotze and some other of these people we don't read also reject modality?

Well, this answer is wrong. Frege and Russell's animus against modality rests on central tenets of their philosophical views. I can tell you a very long story about why the answer is wrong, but I won't, since this paper is about Wittgenstein's bringing modality back into the foundations of logic. Instead, I'll just note a few highlights of that story important for our purposes. If you want the long story, it's in *Necessity Lost* (Shieh 2019).

Frege and Russell, like many contemporary philosophers, take necessity and possibility to be *modes of truth and falsity*, of Fregean thoughts and Russellian propositions. These modes, in turn, are explained in terms of alternatives to the circumstances that actually obtains: a thought might be

true if it is true in alternative, non-actual circumstances, and might be false if it is false in alternative circumstances. Thus, modality require the *relativization of truth and falsity*. A thought or proposition isn't simply true or false. Rather, it's true or false relative to a set of circumstances or a possible world.

Both Frege and Russell insist that there is *no relativization of truth and falsity*. Truth is *absolute*; there is no such thing as truth in some places, at some times, or in certain circumstances, as opposed to truth *simpliciter*. Hence there are no modes of truth, and no such properties as necessity and possibility. *A fortiori*, logic is not explained in terms of these properties.

The crucial question, then, is: why are Frege and Russell opposed to the relativization of truth? Frege's reasons differ from Russell's, but in each case the reasons derive from central philosophical commitments.

The absoluteness of truth for Frege rests on three interconnected positions:

- A sentence that ascribes truth, such as 'it is true that 5 is a prime number', expresses the same thought as the simple sentence '5 is a prime number'.
- Judgment is the acknowledgment of the truth of a thought.
- A "thought" that is not either true or false is not a genuine thought but only an "apparent thought" (*Scheingedanke*); it is not governed by logic.

The claim that truth is relative is the claim that a "thought" (scare-quotes here, of course) is *not by itself* true or false; it is true or false only relative to a circumstance. But then these supposed "thoughts" are not in fact thoughts, but rather at best seeming thoughts. A genuine thought is *by itself* true or false. Hence there is no such thing as relativization of truth or falsity of thoughts.

I will have to go into Russell's views a tiny bit more, since it is by trying to solve the problems that afflict Russell's theory of belief that Wittgenstein started on the path to the *Tractatus*.

When Russell (together with Moore) turned against the British idealism to which he subscribed as an undergraduate, he came to hold a theory of propositions. A proposition is the "*object* of a belief" or a judgment (Moore, 1901: 717). The basic intuition is this. If Socrates and Zeno both judge that Parmenides smiles, then they judge the same thing. It follows that there is an entity which they both judge. That entity is the object of their beliefs. A proposition is *not* a thought or a sentence, *not* a mental or linguistic entity that *represents* a purported fact in the world. Rather, propositions are complex entities whose constituents are parts of the world. A proposition *does not represent* what it's about; it *contains* what it's about.

I now note two features of the Moore-Russell theory of propositions important for our purposes. First, Moore and Russell combines this theory with a rejection of the correspondence theory of truth. Truth is not correspondence to reality, but rather an indefinable property of propositions. The same hold for falsehood; it is not the absence of correspondence with reality, but a primitive property of propositions. Second, the notion of fact, or of the obtaining of a state of affairs in reality, is explained in terms of the truth of a proposition:

[A] fact appears to be merely a true proposition. (Russell 1904: 523)

[W]e must admit that things may really have relations; that their real relations are *facts*, and that these facts are the objects of our judgments when the objects of our judgments are true (Russell 1994: 495).

It is this construal of fact that leads to the absoluteness of truth. Consider first a standard sort of intuition in favor of relativization of truth: a sentence like

Russell is drinking coffee

can be a true description of him at one time, say noon, but a false description at another time, say 1pm. On the Moore-Russell's theory of propositions, this intuition amounts to the claim that the proposition

< Russell, *drinking*, coffee > (1)

has the indefinable property of truth at noon, and the indefinable property of falsity at 1pm. But surely this means that it is a fact that the proposition (1) is true at noon. However, a fact, an obtaining or existing state of affairs, is a true proposition. Thus, according to the proposed account, that (1) is true at noon is a true proposition.

What are the constituents of this proposition? One constituent has to be the proposition (1). One could take the only other constituent to be an indefinable and unanalyzable property, *being true at noon*. But since Russell drinks coffee at times other than noon, (1) is true at those other times as well. So it seems the remaining constituents are the property truth and the time noon. This means that the proposition

< < Russell, *drinking*, coffee >, truth, noon > (2)

has the property of truth. Similarly, according to this account the proposition

< < Russell, *drinking*, coffee >, falsity, 1pm > (3)

has the property of truth. But the truth of the propositions (2) and (3) is *not* relativized to time.

Thus, *fundamentally*, there is no temporal relativization of truth. It *only seems to us* that one and the same proposition, (1), is true at noon and false at 1pm. But what is really the case is the unrelativized truth of *distinct* propositions (2) and (3).

2. *Russell's Multiple-Relation Theory of Judgment*

Starting around 1906, Russell began to have doubts about the Moore-Russell theory of propositions, especially about the idea that there are propositions with a primitive property of falsity. The problem is that a false proposition like that expressed by

Desdemona loves Othello (4)

consists of an entity in which the relation of *love* unites Desdemona to Cassio, and this entity has the unanalyzable property of falsehood. But if the relation of *loves* unites Desdemona to Cassio doesn't this mean that Desdemona loves Cassio? If loving does relate Desdemona to Cassio in this way is it not a *fact* that Desdemona loves Cassio? It seems then that any attempt to specify what a false proposition turns into the specification of something like a fact. But a fact as we saw, is a *true* proposition. (For more on this problem, see Cartwright 1987; Sullivan and Johnston 2018) By 1910, Russell comes to think that the doctrine of false Moore-Russell propositions is untenable: it "leaves the difference between truth and falsehood quite inexplicable" (1910: 152). He goes on to the conclusion that there are no such things as Moore-Russell propositions.

In the Moore-Russell theory, propositions play two roles:

- Objects of belief and other attitudes, which requires that they have the properties of truth and falsity.
- States of affairs in the world, which are parts of the world only when their constituents are unified.

The problem of false propositions shows that states of affairs in the world, consisting of terms unified in some way, are ill-suited to play the role of objects of belief, because we can have false beliefs. This problem leads Russell to the conclusion that the two roles have to be discharged by different things. If there are no propositional entities, then belief or judgment is not a relation to a such entity. What then is belief? Russell's answer is the multiple-relation theory of judgment (MRTJ). Like most theories Russell entertained, the MRTJ went through many versions. I sketch the version criticism of which impelled Wittgenstein to the *Tractatus*.

On the MRTJ, judgment or belief is a state consisting of a subject standing in the belief relation to the objects of her belief. This belief state is true if the objects are indeed connected as the subject believes them to be. It is false if the objects are not so connected. So, when Othello has the false belief that Desdemona loves Cassio, there is no entity in the world consisting of Desdemona standing in the relation *love* to Cassio.

This MRTJ a theory of mental representation by belief or judgment. But it doesn't include a medium of representation. So it differs from linguistic representation. In English a string of words, "some dogs are carnivorous," represents a state of affairs. So the representation is composed of words, and the words are the medium of representation. For Russell, we become acquainted with entities in the world, and then our having a belief is just a matter of our standing in some relation to these entities of acquaintance. There's nothing distinct from the entities of acquaintance whose arrangements somehow represents how the entities are supposed to be arranged.

Along with the MRTJ, Russell comes to adopt a correspondence theory of truth. Truth consists of a relation between particular states or episodes of believing or judging, i.e., of representing the world, and the existence of complexes in the world corresponding to these representing states. These complexes are what Russell's earlier propositions, in their states of affairs aspect, became.

This statement of the theory clearly raises a problem of which Russell is quite aware. What, on the MRTJ, explains the difference between, e.g.,

Othello believes that Desdemona loves Cassio (5)

Othello believes that Cassio loves Desdemona? (6)

Othello might have had both beliefs, but they're different beliefs, since the belief state expressed by (5) is false while that expressed by (6) is true. It is now standard in the secondary literature to call this the "direction problem."

Russell addresses this difficulty by claiming that intrinsic to each (asymmetrical) relation is its *sense*, which means that if two terms stands in that relation, it relates *from one* of the terms *to the other*. Similarly, if it relates three terms, it relates from one to one of the other two and then to the last. And so on. The sense of the relation of believing or judging, then, explains how the belief state expressed by (5) differ from that expressed by (6). In state (5) *belief* relates in the order:

< Othello, Desdemona, *loves*, Cassio > (7)

In state (6) *belief* relates in a different order:

< Othello, Desdemona, *loves*, Cassio > (8)

Wittgenstein's criticism of this MRTJ begins from an examination of the idea that *asymmetrical* relations have *converses*. An asymmetrical relation such as *loves* is one from which more than one complex may be formed together with a single set of terms. For example, the sentence

Desdemona loves Othello (4)

expresses a different complex from

Othello loves Desdemona (9)

Given the doctrine of the sense of relations, the difference between these complexes consists of the relation *loves* relating Othello and Desdemona in different orders.

From *The Principles of Mathematics* (1903) onwards, Russell tends to hold that for each asymmetrical relation there is a distinct relation that is its *converse*. The converse of *loves* is the relation expressed by 'loved by' such that when and only when x loves y , y is loved by x . If converses are distinct from the relations of which they are converses, then the complexes expressed by

Desdemona loves Othello (4)

and

Othello is loved by Desdemona (10)

are distinct. Wittgenstein convinced Russell that this consequence is implausible: in reality there is surely only one fact consisting of Desdemona loving Othello. Thus, in *The Theory of Knowledge*, written as Wittgenstein urged this criticism, Russell adopts the view that the sentences (4) and (10) express exactly the same complex. 'Is loved by' *does not denote a distinct "converse"* relation of the relation denoted by 'loves'. Rather, 'is loved by' and 'loves' *denote the same* relation. In reality, there are no distinct converse relations of asymmetrical relations.

One further claim tacitly accepted by Russell leads to the downfall of the MRTJ: the *left-to-right orders of words* in sentences such as (4) and (10) reflect the order in which the relations in question relate their terms into a complex. So

Desdemona loves Othello (4)

describes a complex in which *loves* relates *from* Desdemona *to* Othello.¹

We can now articulate Wittgenstein's criticism of the MRTJ as a dilemma for Russell resulting from four commitments. Russell accepts:

1. Multiple complexes come from an asymmetrical relation and a single set of terms.

¹My reading of Wittgenstein's criticism of the MRTJ is based on Ricketts (1996). Russell initially replies to the difficulties engendered by asymmetrical converse with a theory of position relations, see (1913); for an account of how this theory falls apart, see Pincock (2008). I complement Pincock's account with an analysis of the modal aspect of the trouble Russell faces in (forthcoming). For a venerable alternative view of Wittgenstein's criticism, see Griffin (1985). Landini (2007) is another useful alternative. For a non-historical discussion of the philosophical issues raised by relations and their "converses," see Fine (2000).

2. There are no converses of relations.

But these are incompatible with:

3. Relations have sense

4. The order of words in a sentences gives order of relating

For example, the names 'Othello' and 'Desdemona' occur in (4) and in (10) in *different left-to-right orders*. Because linguistic order reveals order in relating, this implies that the single relation expressed by these sentences relate Othello and Desdemona in different orders. It follows that these sentences describe *different* complexes composed of Desdemona, Othello, and *loves*. But according to Russell's view of converse relations, they express the *same* complex. So one of 1- 4 has to go.

Russell lets go of 3, the sense of relations. This is disastrous for the MRTJ. If there is no order in relating, then Russell loses his solution to the direction problem.

3. Wittgenstein's "Notes on Logic"

Wittgenstein undertakes to solve the problems of MRTJ in a set of remarks dictated to a stenographer in Manchester and to Russell in Cambridge, now called the "Notes on Logic" (1913: hereafter *NL*). I will refer to them in the form x - y for manuscript x , remark y .

Wittgenstein reinstates propositions as objects of belief. But, he take propositions to be made up of different things from what the proposition represents. That is to say, he introduces a medium of representation.

More importantly, Wittgenstein takes propositions to be *facts*:

Propositions [which are symbols having reference to facts] are *themselves facts*: that this inkpot is on this table may express that I sit in this chair. (*NL*: 1-2; emphases mine)

Propositions, Wittgenstein claims, have *sense*:

Every proposition is essentially true-false: to understand it, we must know both what must be the case if it is true, and what must be the case if it is false. Thus a proposition has two *poles*, corresponding to the case of its truth and the case of its falsehood. We call this the *sense* of a proposition. (*NL*: S-13).

I understand the proposition ' aRb ' when I know that either the fact that aRb or the fact that not aRb corresponds to it. (*NL*: 4-6)

This seems to say that the sense of a proposition ' aRb ' is a *pair* of facts—the fact that aRb and the fact that not aRb —and these facts are the *poles* of the proposition.

Moreover, propositions have *meaning*:

[W]e can only know the *meaning* of a proposition when we know if it is true or false. (*NL*: 3-25; emphases mine)

Meaning is somehow connected with *negation*:

In my theory p has the *same meaning* as not- p but *opposite sense*. The meaning is the fact. (NL: S-20; emphases mine; see also 4-2)

[I]t is important that we *can* mean the same by ' q ' as by 'not- q ', for it shows that neither to the symbol 'not' nor to the manner of its combination with ' q ' does a characteristic of the denotation of ' q ' correspond. (NL: 1-9)

Wittgenstein also talks of *positive* and *negative facts*:

There are positive and negative facts: if the proposition 'this rose is not red' is true, then what it signifies is negative. But the occurrence of the word 'not' does not indicate this unless we know that the signification of the proposition 'this rose is red' (when it is true) is positive. It is only from both, the negation and the negated proposition, that we can conclude to a characteristic of the significance of the whole proposition. (NL: 1-7).

It's less than clear what this all means. Here are some obvious questions:

- The fact that aRb is a positive fact; is the fact that not- aRb negative?
- How is it that a proposition and its negation have the same meaning but opposite sense?
- It seems Wittgenstein is saying this: whether what corresponds to a given proposition, if true, is a positive or a negative fact is not determined independently of whether the negation of that proposition, if true, corresponds to a positive or a negative fact. Why?

To answer these puzzles, I turn to a well-known and perhaps even more puzzling text from NL:

[T]he form of a proposition symbolizes in the following way: Let us consider symbols of the form " xRy "; to these correspond primarily pairs of objects, of which one has the *name* " x ", the other the *name* " y ". The x 's and y 's stand in various relations to each other, among others the *relation* R holds between some, but not between others. I now determine the sense of " xRy " by laying down: when the facts behave in regard to {editors' note: *sich verhalten zu*, are related to} " xRy " so that the *meaning* of " x " stands in the relation R to the *meaning* of " y ", then I say that the[y] [the facts] are 'of like sense' ['gleichsinnig'] with the proposition " xRy "; otherwise, 'of opposite sense' ['entgegengesetzt']; I correlate the facts to the symbol " xRy " *by* thus dividing them into those of like sense and those of opposite sense. To this correlation corresponds the correlation of name and meaning. Both are psychological. Thus I understand the form " xRy " when I know that it discriminates the behaviour of x and y according as these stand in the relation R or not. (NL: 4-8; emphases mine; all square brackets in original)

I contend that this passage shows that central to Wittgenstein's theory are four claims:

- Facts have *forms*
- A fact represents another fact through the *symbolizing of the form* of the first fact
- The symbolizing of a form depends on a *stipulation*.
- The stipulation fixes the *sense* of the symbolizing form.

The “symbols” in this text are propositions. That is to say, they are facts. For Russell facts are just entities with parts, composed of those parts. For Wittgenstein, in contrast, a fact is an *aspect* of a collection or a composite. The form of a propositional fact is something that it has in common with other propositional facts. Let me now illustrate these ideas with some examples.

The first example is a photograph:



There are many facts about this photograph, for example:

1. the (image of the) panda eating the apple is to the (audience-)left of the panda not eating
2. both pandas have one hand resting on a leg
3. there is grass behind the pandas, and grass in front of them.

Keep in mind the first of these facts.

The second example is the following sequence of Latin alphabet letters and blank spaces:

Desdemona loves Othello

Let's focus on the following facts about this sequence of letters and blanks:

4. The word 'Desdemona' occurs to the left of the word 'loves'
5. The word 'Cassio' occurs to the right of 'loves'.

The third example is a non-Latin sequence of letters and blanks:

عطیل یحب سے محبت کرتا ہے

Here is a fact about this sequence:

6. The sign 'یحب' occurs to the left of the sign 'عطیل' and the sign 'سے محبت کرتا ہے' occurs to the left of the sign 'یحب'.

The fourth example a sequence of characters:

苔
丝
狄
蒙
娜
爱
奥
赛
罗

- the left side panda stands for Desdemona,
- the right side panda stands for Othello, and
- the fact that *Othello loves Desdemona* makes fact (12) true.

Under this alternative stipulation, exactly the same fact (12) about the photograph of the pandas represents a different fact about the two people, Desdemona and Othello.

Wittgenstein describes these stipulations as specifying *how* a proposition is *compared* with facts. Propositions are *not only* facts, they are fact *together with* a stipulated way of *comparison*.

Now we can specify how Wittgenstein's theory in *NL* dissolves Russell's dilemma. On this theory, the difference in the left-to-right order of names in

Desdemona loves Othello (4)

Othello is loved by Desdemona (10)

doesn't *by itself* imply that these propositional facts represent different facts about how Desdemona and Othello stand with respect to the relation *loves*. This is because what fact is represented by a propositional fact depends

- not *merely* on the order of names in the representing fact,
- but also on the *stipulation* of what facts make the representing fact true.

The explanation of why (4) and (10) represent the same fact has two factors. First, (4) and (10) both have the form xRy . Second, different stipulations specify what facts make (4) and (10) true. The stipulation for (4) is:

The fact that the entity whose name is to the *left of 'loves'* loves
the entity whose name is on the *right of 'loves'* (13)

makes (4) true. The stipulation for (10), by contrast, is this:

The fact that the entity whose name is to the *right of 'is loved by'* loves
the entity whose name is on the *left of 'is loved by'* (14)

makes (4) true.

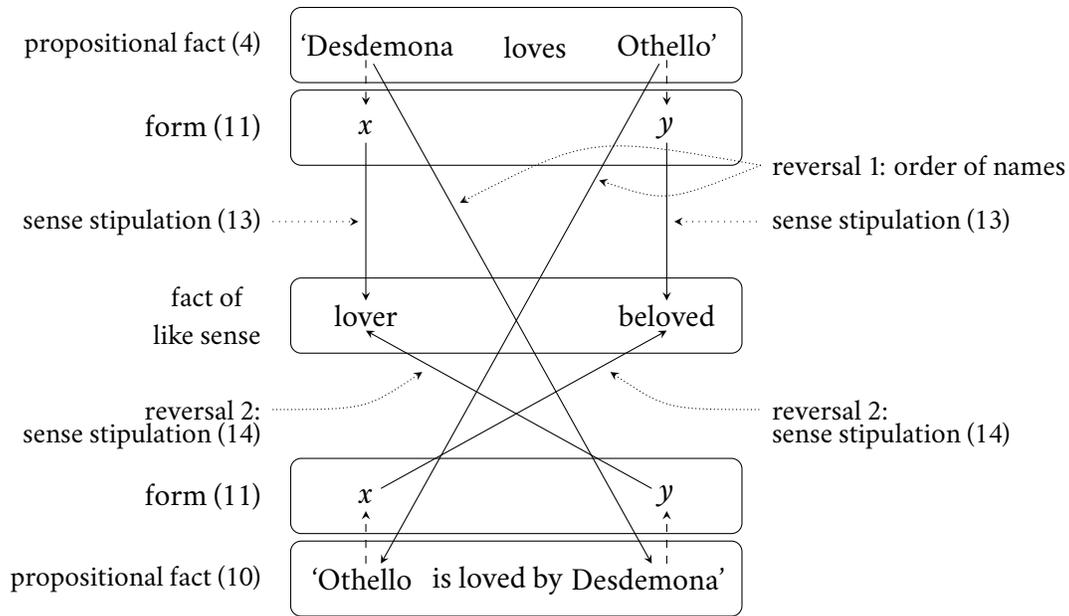
We can also see why (4) and (10) represent the same fact. These *stipulations reverse* which entity is the lover and which the beloved in the fact that makes the proposition true.

Wittgenstein can now also account for the phenomena that lead us to think that there are converses of asymmetrical relations. Here is why

Desdemona loves Othello (4)

Othello is loved by Desdemona (10)

represent the same fact. Although the left-to-right order of names in (4) is the reverse of the order of names in (10), this reversal is *canceled* out by the *reversal* of the lover and beloved in the stipulations:



To sum up, Wittgenstein's *NL* theory in essence *rejects*

- Order of words gives order of relating, and
- More generally, the structure of a representing fact doesn't transparently yield the structure of the fact represented.

Wittgenstein also rejects:

- An asymmetrical relation is different from its converse.

Wittgenstein is then able to hold on to:

- Relations have sense.

So far we have only talked of stipulations of what facts are of "like sense" with a propositional fact. Clearly, though, Wittgenstein thinks that the symbolizing of forms of facts also involve stipulations of what facts are of "opposite sense" with the propositional fact. Wittgenstein's theory here is bound up with his notion of negative facts and his theory of negation.

Consider again the stipulation for (4):

The fact that the entity whose name is to the *left of 'loves'* loves
the entity whose name is on the *right of 'loves'*

makes (4) true. Let's call this fact F^+ .

This stipulation is completed by the further "opposite sense" clause that

The fact that the entity whose name is to the *left of 'loves'* does not love
the entity whose name is on the *right of 'loves'*

makes (4) false. Let's call this fact F^- . This further clause stipulates a negative fact, a fact that some entity does not stand in a relation to another entity, as that which make a propositional fact false.

Let's say that the positive fact F^+ that makes (4) true and the negative fact F^- that makes (4) false together form a *matched pair* of facts. The stipulation that we have just set out clearly reflects the standard meaning of (4) in English. Let (\dagger) be the name of this stipulation. And let's say that (\dagger) is the *conventional sense-stipulation* for (4).

Wittgenstein's account of negation rests on the intuitive idea that if a proposition is true then its negation is false, and *vice versa*. Let p be a proposition consisting of a fact of form $\lceil x \text{ loves } y \rceil$ together with some sense-stipulation. The intuition suggests that the negation of p should be a proposition q that satisfies the following condition:

$$\begin{aligned} q \text{ is of like sense with the fact that is of opposite sense with } p, \text{ and} \\ q \text{ is of opposite sense with the fact that is of like sense with } p \end{aligned} \tag{*}$$

I now describe a sense-stipulation, (\ddagger) , for (4) different from (\dagger) :

- the negative fact F^- makes (4) true
- the positive fact F^+ makes (4) false.

Obviously (\ddagger) reverses the like-sense and opposite-sense facts of stipulation (\dagger) . Let's call these two stipulations "of opposite sense" to one another.

Now, suppose that p is the proposition consisting of the fact

$$\text{Desdemona loves Othello} \tag{4}$$

together with stipulation (\dagger) . Then, the condition (*) for being the negation of p is satisfied by the fact (4) together with stipulation (\ddagger) .

That is to say, the fact

$$\text{Desdemona loves Othello} \tag{4}$$

together with stipulation (\ddagger) , is the negation of the proposition that consists of this very fact together with the conventional stipulation (\dagger) .

In most natural languages negation is formed by adding some expression of negation to a sentence— 'not', '不', 'ليس', etc. So we normally take the negation of (4) to be

$$\text{Desdemona does not love Othello} \tag{15}$$

But from Wittgenstein's perspective what is important is the relationship of opposition between sense stipulations rather than the propositional facts. So 'not' is not required for negation.

Moreover the occurrence of the word 'not' in (15) does not make it, if true, signify a negative fact unless (4), if true, signifies the matching positive fact. Nothing stands in the way of giving (4) the sense stipulation (\ddagger) , opposite to the conventional one, and (15) the conventional stipulation (\dagger) . In this case we might say that (4) is the negation of (15), formed by omitting 'not' from the propositional fact (15).²

²The reader may profit from the mostly historical discussion of "Notes on Logic" in Potter (2009).

4. *The Truth-Problem in Wittgenstein's Wartime Notebooks*

Although the *NL* theory resolves Russell's difficulty with the MRTJ, Wittgenstein came to realize that it does *not* provide a satisfactory account of *falsity*. The realization came to him when he was serving as an officer in the Austrian army in World War I. During this time he kept a number of notebooks, now published in *Notebooks 1914-1916* (1979: hereafter *NB*), in which he recorded his thoughts for a book on logic and philosophy—which of course became the *Tractatus*. In the first of these notebooks, in Sept 1914, Wittgenstein discusses something he calls the "truth-problem" (*Wahrheitsproblem*).

According to *NL*, the sense of a proposition *P* is fixed by stipulating a fact *F* as "like sense" with a propositional fact *Now*, if the proposition *P* is *false*, then presumably the fact *F* either isn't the case or doesn't obtain. But what does it mean to say that a fact isn't the case?

If the proposition

Abelard loves Heloise

is false, it surely doesn't describe any aspect of the world. Doesn't this mean that there is *no fact that corresponds to it*? What, then, has the sense stipulation correlated with the propositional fact?

What if we just bite the bullet, and insist that what makes

Abelard does not love Heloise

true is an absence of any fact that corresponds to

Abelard loves Heloise?

But now consider another true negative proposition,

Iago does not love Cassio.

What makes this proposition true, on the view that we're considering, is the absence of any fact corresponding to

Iago loves Cassio

Now, the absence of a fact is not a feature of the world, not something obtaining in the world. So it's not clear how

- the absence of any fact corresponding to 'Abelard loves Heloise'
- the absence of any fact corresponding to 'Iago loves Cassio'

are different. But they have to be different, because

- the first absence wouldn't make the proposition that Iago doesn't loves Cassio true,
- the second absence wouldn't make the proposition that Abelard doesn't love Heloise true.

These puzzles problematize the notion of negative fact from *NL*. Consider F^- , the supposed fact that some entity does not stand in the relation *love* to another entity. Isn't this just the *failure to obtain of the positive fact* F^+ with which F^- forms a matched pair? Is it then an absence of from the world? Hence Wittgenstein's agony:

It is the dualism, positive and negative facts, that gives me no peace. For such a dualism can't exist. But how to get away from it? (NB: 24/11/1914)

Without negative facts, the matched-pair theory of negation from *NL* collapses.

Wittgenstein's truth-problem is a version of an ancient problem of falsity: how is it possible to "say, speak, or think *that which is not* itself correctly by itself?" (Plato 1985: 238c). Wittgenstein struggled with the problem of falsity throughout the first wartime notebooks. Let's now look at some of these struggles.

5. *The First Picture Theory*

One of the most famous aspects of the *Tractatus* is the *picture theory of propositions*. It is in trying to solve the problem of falsity that Wittgenstein hit upon the idea that propositions may be taken to be *pictures* or, more precisely, *models*:

In the proposition a world is put together experimentally. (As when in the law-court in Paris an automobile accident is presented by means of dolls, etc.)

This must yield the nature of truth straight away (if I were not blind). (NB: 29/9/1914)

What strikes Wittgenstein about pictures is that they "can be right and wrong:

Let us think of hieroglyphic writing in which each word presents its meaning! Let us think also of the fact that actual pictures of facts can be *right* and *wrong*.

: If the right-hand figure in this picture represents the man A, and the left-hand one signifies the man B, then the whole might assert, e.g.: 'A is fencing with B'. The proposition in picture-writing can be true and false. (NB: 29/9/1914)

What is it for a models of accidents, actual pictures, and hieroglyphic writing to be *wrong*? It is for them to present something, say the holding of a relation, that doesn't exist:

A picture can present relations that do not exist!!! (NB: 30/9/1914)

Wittgenstein immediately asks,

How is that possible? (NB: 30/9/1914)

He realizes that the analogy of proposition and picture by itself is merely suggestive. It provides no solution to the problem of falsity unless we show how pictures can be false in virtue of representing what doesn't exist.

The first step Wittgenstein takes towards a solution is to appeal to the idea of form from *NL*: a picture or proposition must have something in common with the world, if it is to model the world, rightly or wrongly. Propositions presuppose that facts have a "logical structure":

[I]n order for a proposition to be CAPABLE of having SENSE, the world must already have just the *logical structure* that it has. The logic of the world is prior to all truth and falsehood. (NB: 18/10/1914)

One must gather from the proposition the *logical structure of the facts* that makes it true or false. (NB: 20/10/1914; emphasis mine)

Pictures have “form,” and this form is something in the picture that is “identical with reality”:

The *form of a picture* might be called that in which the picture *MUST agree with reality* (in order to be capable of portraying it at all). (NB: 20/10/1914; emphasis mine)

The theory of logical picturing through language says—quite generally: In order for it to be possible that a proposition should be true or false—agree with reality or not—for this to be possible *something in the proposition* must be *identical with reality*. (NB: 20/10/1914; emphasis Wittgenstein’s)

So the form of a proposition is identical with the logical structure of that fact that the proposition pictures.

But what is this common form? The idea of common form or structure may be based on what Wittgenstein read about the representation of traffic accidents in Paris law-courts:

that one doll is placed to the left of a block, for example, might model the fact that Michel was standing on the left of Jean-Pierre’s Renault. In such cases the spatial relation *being to the left of* in which the doll and the block stand to one another show the people and car as standing in this very same spatial relation.

In this particular case there seems to be a fairly strict identity of structure or form, consisting in the same spatial relation holding in the model and in the represented fact. (But is it really the *same* spatial relation? Isn’t one, e.g., *being to the left by five centimeters* while the other is, e.g., *being to the left of by five meters*?)

In general the identity of structure can’t be this strict. For example, one can use the relation *being to the right of* among model elements to depict *being to the left of* among the things modeled. Here the relations are both *spatial*. So is it that picture and fact have the same form because the structure of the picture and the structure of the fact are both *spatial*?

This obviously is also wrong. Speech models situations, but the relations among spoken words are temporal. So in this case the *spatial* relation *being to the left of* might be modeled by the use of a *temporal* relation. So exactly what in the proposition is identical with reality is very unclear. All we can say is that some aspect of the way in which a proposition is put together is the same as some aspect of how a fact that makes that proposition true is put together.

Wittgenstein takes the idea of some identity of structure or form to address the truth-problem:

The first thing that the theory of logical picturing through language gives us is information about the nature of the truth-relation. (NB: 20/10/1914)

But how?

According to the theory of propositions as pictures:

What, in the first instance, corresponds in the world to a proposition is a structure or form. This structure is in some sense identical to the structure of the proposition. The correctness or incorrectness of the proposition is then determined by whether there exists a fact with that structure.

So now the truth and falsity of propositions do not consist in the obtaining of a correlated fact. Rather, a proposition corresponds to (a structure related to) the structure it instantiates,

if it is false, that is because in the world there is *no* fact of that structure, *not* because there exists some fact having the special characteristic of not obtaining.

We no longer need to explain

what is a fact that doesn't obtain, or how absences of facts are distinguished from one another.

But Wittgenstein immediately starts to doubt this attempted solution to the truth-problem:

That in a certain sense the logical form of p must be present, even if p is not the case, shows itself symbolically through the fact that ' p ' occurs in ' $\sim p$ '.

This is the difficulty: How can there be the form of p , if there is no fact of this form? And in that case, what does this form really consist in?! (NB: 29/10/1914)

The problem comes from Wittgenstein's remaining allegiance to Russell's ideas. For Russell *form* is what is common to a class of complexes, so there is no form if there are no complexes. Wittgenstein's worry, then, is that the forms of facts are dependent on the facts which have the form, so if certain facts don't exist, then their form don't exist either.

In early November Wittgenstein moves in another direction. He again brings up the now-familiar issue of falsity:

How does the picture represent a fact?

It is after all itself *not the fact*, which *need not be the case at all*. (NB: 4/11/1914; emphases mine)

Then he answers,

One name represents one thing, another another thing, and they themselves are connected; thus the whole—like a *tableau vivant*—represents the fact.

The logical connection must of course be *possible* for the represented things (NB: 4/11/1914; first emphasis mine)

The idea is that what in reality correspond to pictures are *not existing entities*:

- a fact, a connection of things, or a way in which things are connected

But rather possibilities:

- possible connections of things, or possible ways for the things to be connected.

This idea, and the truth-problem, are not taken up again in the notebooks. However, as we'll see, in the *Tractatus* *possibility* becomes central.

6. *Possibility in the Tractatus*

A number of doctrines from *NL* and *NB* reappear in the *Tractatus*:

- Propositions are facts.
- Propositions are pictures of facts.
- A proposition pictures a fact by having a form that the fact also has.
- The truth and falsity of propositions result from a comparison of propositional facts with what they picture, namely, facts in the world.

However, the two main concepts appearing in these doctrines:

- Fact
- Form

are in the *Tractatus* based on a primitive notion of possibility. So in the *Tractatus* the meanings of these doctrines are transformed.

We start with the notion of fact from early in the *Tractatus*, in its famous opening:

- 1 The world is everything that is the case.
- 1.1 The world is the totality of facts, not of things.
- 2 What is the case, the fact, is the existence of *states of things*.
- 2.01 A state of things is a *connection of objects (entities, things)*.
- 2.03 In a state of things objects hang one in another, like the links of a chain.

These remarks suggest that a state of things is something like this:



That is to say, a state of things is like a physical object made up by connecting other physical objects together. Now, intuitively, each of the links is an object in its own right, independent of being connected with the other links.

But this is altogether the wrong conception of object and state of things. Let's start with the notion of object :

- 2.011 It is essential to a thing that it can be a constituent part of a state of things.
- 2.0121(4) Just as we cannot think of spatial objects at all apart from space, or temporal objects apart from time, so we *cannot think of any object apart from the possibility of its connection* with other things.
If I can think of an object connected in a state of things, I cannot think of it apart from the *possibility* of this connection.

Objects are *not independent* of states of things, as the link of a chain is independent of that chain. Rather, an object is defined by its *possible occurrences* in state of things.

But states of things are equally not prior to objects:

2.0124 If all objects are given, then thereby are all *possible states of things* also given.

2.014 Objects contain the *possibility of all situations*.

What states of things are possible, it seems, depends on what objects there are, that is to say, on what *possibilities for combining into states of things* there are.

Wittgenstein tries to clarify this by an analogy with physical space:

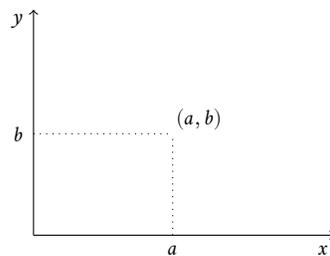
2.013 Every thing is, as it were, in a space of possible states of things. I can think of this space as empty, but not of the thing without the space.

in and ask, what does it mean to say that spatial objects are not thinkable apart from space? For Wittgenstein the answer is that spatial objects are *essentially occupiers of spatial locations*, and so essentially spatially related to other spatial objects.

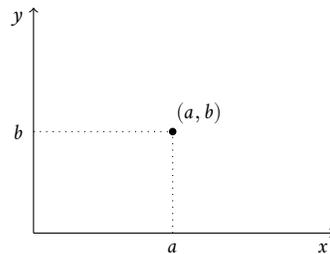
This suggests that a Tractarian object essentially “occupies” a possibility of connection with other objects. But in what sense does an object “occupy” a possibility of connection? Answer: the “occupation” of a possibility is the *realization* of that possibility.

A *possibility of connection* of objects is a *logical place*. Objects occupy that logical place just in case that possibility of connection is realized.

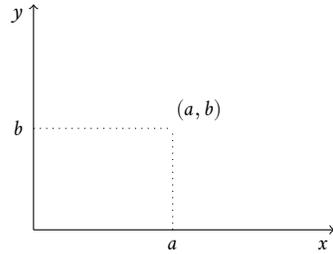
Here is a simplified model (following Sullivan 2001) of the space of possibilities as a two dimensional plane with Cartesian coordinates. Each point on the x - and the y -axes represents an object. If the Cartesian coordinates of a point is (a, b) , then this point represents the *possibility* that the objects a and b are combined in a state of things.



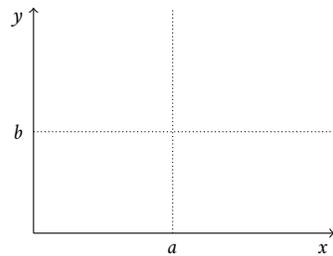
The point (a, b) is *filled in* if a and b are combined in a state of things.



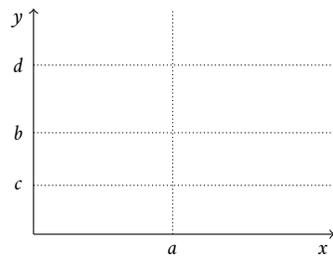
The point (a, b) is *blank* if a and b are *not* combined in a state of things.



In this model, an *object* can be thought of as a *vertical or horizontal line*:



The vertical line is the object *a*; the horizontal line is the object *b*.



Along the vertical line that is *a* are all the points that can be occupied in virtue of *a*'s combining with some object on the *y*-axis, i.e., all the possible combinations of *a* with objects. If the object *a* is the entire vertical line, then it *contains* all the points of its intersection with other objects

Each point of intersection is a possible combination of *a* with that object. So, *literally*, all the possibilities of *a*'s occurring in a state of things are parts of *a*. It is *essential to a* that it intersects with all the horizontal lines. So it is essential to *a* that it contains all the possibilities of its combining with other objects. Possibilities of states of things involving finitely many objects are finite-dimensional spaces; objects are hyperplanes in such spaces.

This way of taking the model helps in thinking about

2.03 In the atomic fact objects hang one in another, like the links of a chain.

The immediate point of this remark is to suggest that when objects are combined, there isn't, as it were, something, some relation, that holds them together, just as there isn't anything other than the links that hold the links together. So, in particular, Wittgenstein is here rejecting Russell's view that in complexes there is a relating relation that does the job of unifying the terms into that complex. Now we can go a step further: not only is there no further entity that unifies objects, but it is part

of the very identity of each object that it can combine with other objects. This Russellian connection also allows us to clear up an exegetical debate. There are two apparently opposed views about whether objects are all particular, or whether they include also properties and relations. But there need be no real opposition between these views. What Wittgenstein is denying, in Russellian terminology, is that any object is a relating relation. We may take objects to include individuals and relations, so long as it is not held that some relation is the entity that unifies the remaining objects into the combination.

Wittgenstein draws an important contrast between *structure* and *form*:

- 2.031 In a state of things objects stand to one another in a determinate way.
- 2.032 The way in which objects hang together in a state of things is the *structure* of the state of things.
- 2.033 *Form* is the *possibility of structure*. (Emphasis mine)

This distinction was added by Wittgenstein in the final stages of the composition of the *Tractatus*; it is not present in the *Proto-Tractatus*. In order to understand this contrast, one has to see that there are two concepts of structure.

A structure may be the way in which a composite entity is put together, so that distinct composites can be put together in the same way. This is something like a pattern that can be instantiated by various entities. Alternatively, one sometimes talks of a building as a structure. Here “structure” means a composite entity.

One *can* think of a picture or a state of things as a structure in the second sense, that is, as a composite entity. But in the *Tractatus* the structure of a state of things is structure *in the first sense*. Two states of things, two connections of objects, can have the same structure if in each connection the objects stand to one another in the same determinate way.

One difficulty some may experience with the Tractarian structure/form distinction is that ‘form’ is sometimes used to mean the way in which some composite is put together. This is very close to Russell’s notion of form: the way in which a complex is put together out of terms. If one understands form in this way it seems to make no sense to distinguish it from structure.

Consider, however, Russell’s own description of the problem posed by asymmetrical relations: “with a given relation and given terms, two complexes are ‘logically possible’” (1913: 111). That is to say, he accepts that, *prima facie*, there are two *possible ways* in which a complex can be constituted from these ingredients. This means two possible complexes with *distinct possible structures*. For Russell the notion of possibility has to be analyzed away; not so for Wittgenstein. The notion of a *possible “way” of composing* is fundamental; in particular, it is independent of whether there exists any composites realizing it. Wittgenstein’s name for this notion is *form*.

The significance of the distinction is that a *state of things* is the *realization* or the *obtaining* of a *possibility*. A state of things exists just in case a possibility of things standing to one another in a determinate way is realized. It doesn’t exist if that possibility, that form of determinate connection, is not realized.

Form brings Wittgenstein peace on positive and negative facts:

- 2.06(2) (The existence of states of things we also call a positive fact, their non-existence a negative fact.)

Part of the truth-problem is that negative facts are not distinguishable from the absence of fact. In the *Tractatus* the notion of possibilities for things to “connect” into facts, and the notion of a possibility realized or not realized, obtaining or not obtaining, are primitive. One can then take negative states of things to be unrealized, i.e., non-existent, possibilities. There is then no problem of how to distinguish them from absence of fact; they are absences of facts, but absences whose specificities are founded on distinctions among possibilities.

7. *Picturing in the Tractatus*

We can now begin to understand the view of propositions as pictures in the *Tractatus*. A “picture is a fact” (2.141). It is a fact “that [the] elements [of the picture] stand to one another in a determinate way” (2.14). These pictorial elements “correspond” to “objects” (2.13) and “are representatives, in the picture, for objects” (2.131). 2.15-2.17 then provide an account of how these features of a picture relate to its being “a model of reality” (2.12).

In 2.15 Wittgenstein draws a structure/form distinction for pictures:

- 2.15(1) The ... elements of a picture *stand to one another in a determinate way* ...
- 2.15(2) This *hanging together of the elements* of the picture is called its *structure* [T]he *possibility of this structure* is called its *form of depiction*.

What makes a picture represent is the structure it has:

- 2.15 The fact that the elements of a picture stand to one another in a determinate way represents that things stand so to one another.

This means that a pictorial fact “represents” that things stand to one another in the *same determinate way* as pictorial elements do. A picture represents this by means of the *form of depiction*:

- 2.151 The form of depiction is the *possibility* that things stand to one another as do the elements of the picture.

Picturing works like this. The pictorial elements stand to each other in a determinate way. This is what it is for a picture to be a fact; it is an *existing state* of pictorial elements. This state is a *realization of the possibility* that pictorial elements are connected in this determinate way, a realization of a form.

If this form is depicting form, then it is also *possible* for the things correlated with pictorial elements to be connected in a state of things with the *same* structure as the picturing fact. The possibility that pictured things stand to one another in this determinate way is the possibility that they are connected in a state of things.

Thus, in virtue of this identity of possibility of connection, a picture *presents* a *possible* existence of a *state of things*:

- 2.201 The picture depicts reality by *presenting a possibility* of the *existence ... of states of things*.

What a picture *represents* is *not just* this possible existence of a state of things; it represents this possibility *as obtaining*.

A propositional picture is true if the possibility it presents as obtaining is realized, false otherwise. This conception makes room for false propositions as propositions presenting possibilities that are not realized in the world. Thus, in the *Tractatus* it is by the incorporation of modality into the nature of picturing that Wittgenstein finally solves the problem of false propositions. So for Wittgenstein the very notions of a proposition and of its truth or falsity are intrinsically modal.

We have just seen that the truth and falsity of propositions are understood in terms of possibilities. So, in contrast to Frege and Russell, Wittgenstein does not conceive of possibility as a mode of truth. Possibility is primitive and prior to truth and falsity. It is only in virtue of presenting possibilities that propositions are capable of being true or false.

To sum up, the conception of picturing in the *Tractatus* is not the same, but is a development of the common structure/form idea of *NB*. In the *NB* theory the facts of *NL* are replaced with structure/form as the fundamental correlate in reality of propositions. In the *Tractatus* Wittgenstein takes the final step merely considered in *NB*: form is an irreducible possibility. Thus, in the *Tractatus*, what is fundamental to pictures and propositions is *not* any part of reality, *but* rather the *possibilities for being part* of reality.

At the end of this road to a solution to the Truth-Problem, modality has been brought back into the nature of propositions. And so modality is now back in logic, which governs propositions and their truth or falsity, or rather, permeates their being. Naturally the defense of this last qualification will have to await another occasion.

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